Lesson Activity 1

1.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

2. If an organism eats fish, then it is an eagle.; No, people eat fish, but they are not eagles.

3. The cases where $p \rightarrow q$ is true yields this table. The fourth column shows the resulting values for $q \rightarrow p$. As a result, the converse of $p \rightarrow q$ is not always true.

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
F	T	T	F
F	F	T	T

4. The cases where $p \rightarrow q$ is true yields this table.

p	q	$p \rightarrow q$
T	T	T
F	T	T
F	F	T

Use it to generate this truth table. This shows that if $p \rightarrow q$, then $\neg q \rightarrow \neg p$ is also true.

$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
F	F	T
F	T	T
T	T	T

5. In all cases the statement $p \vee \neg p$ is true.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

6. In all cases the statement $p \wedge \neg p$ is false.

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

7.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

8. **a.** Let p = an organism has flight feathers and q = it can fly. The logic statement is $p \rightarrow q$.

b. No. There are flying organisms that aren't birds. For example, bats.

c. Yes. Any organism that cannot fly doesn't have flight feathers.

d. No. Since the converse isn't true, the statement is not bi-directional.

9. **a.** Let p = has contour feathers, q = has flight feathers, and r = it can fly. The logic statement is $(p \wedge q) \rightarrow r$.

b.

p	q	r	$(p \wedge q) \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	F	T	T
F	T	F	T
F	F	F	T

c. Neither

10. **a.** Let p = has contour feathers, q = has flight feathers, and r = it is a bird. The logic statement is $(p \vee q) \rightarrow r$.

b.

p	q	r	$(p \vee q) \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	F	T	T
F	T	F	F
F	F	F	T

c. Neither

11. Check students' work.

Investigation Practice 1

a. **1.** The statement is also true.

2. It is the contrapositive of the original statement

b. **1.** Neither.

2. Tautology

3. Tautology

c. **1.** Equivalent

2. Not equivalent

Lesson Activity 2

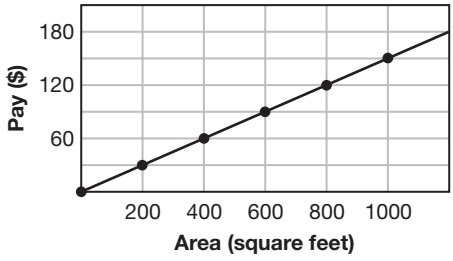
1. a. 1400

   b. $210

2.

t (hours)	0	1	2	3	4	5
x (square feet)	0	200	400	600	800	1000
y (dollars)	0	30	60	90	120	150

3.



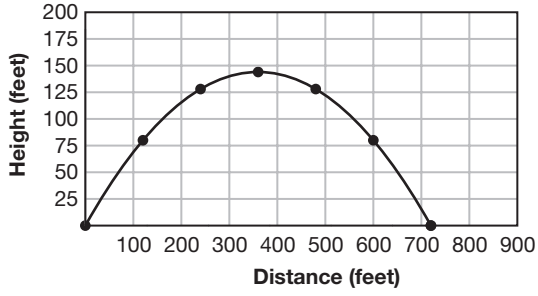
4. The graph is a straight line with a positive slope. The point (200, 30) shows that the painter earns $30 for painting 200 ft^2.

5. d. The graphs are the same.

6.

t (seconds)	0	1	2	3	4	5	6
x (feet)	0	120	240	360	480	600	720
y (feet)	0	80	128	144	128	80	0

7. The graph is a parabola that opens downward.



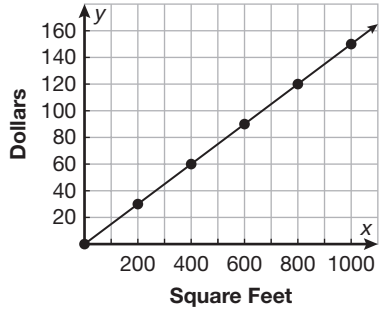
8. d. The golf ball reaches its highest point, 144 feet high, in 3 seconds. It takes the golf ball 6 seconds to reach the ground, and it travels 720 feet.

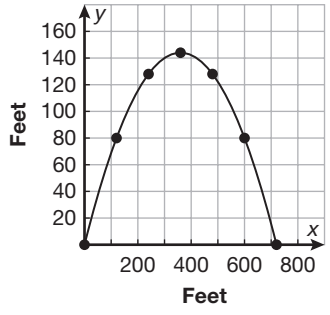
Investigation Practice 2

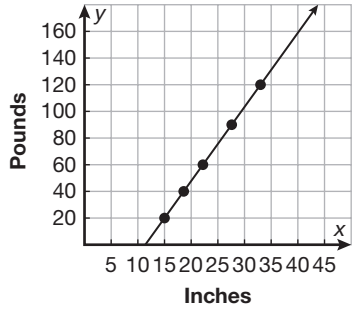
a.

t **(months)**	2	4	6	9	12
x **(inches)**	15	18.6	22.2	27.6	33
y **(pounds)**	20	40	60	90	120

b.



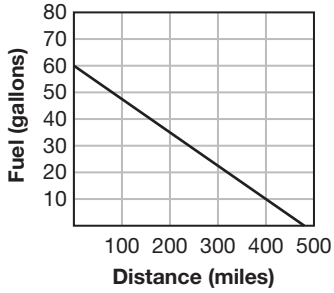




c. 70 pounds

d. $x = 40t$; $y = 60 - 5t$

e. The graph is a straight line with a negative slope.

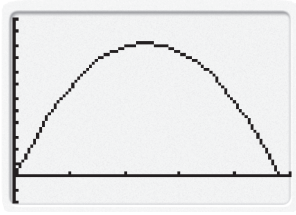


f. The bus can travel 480 miles on one tank of fuel. It can be driven 12 hours before refueling.

g. $x = 60t$

$y = -16t^2 + 128t$

h.



i. The *x*-coordinate gives the horizontal distance the ball has traveled at a given time; the *y*-coordinate gives the height of the ball at that time.

j. The golf ball reaches its highest point, 256 feet high, in 4 seconds. It takes the golf ball 8 seconds to reach the ground, and it travels 480 feet.

k. The second shot had a greater vertical velocity than the first, so it went higher. The first shot had a greater horizontal velocity than the second did, so it traveled a greater horizontal distance.

Lesson Activity 3

1. $y = x + 4$

2. The resulting expression is substituted into the other equation; $2x - (x + 4) = -10$

3. Sample: Forming a one-variable equation produces a solvable equation

4. Solve for x in the equation from step 2, which is $x = -6$. Then substitute the value for x into one of the original the equations, which is $6 + y = 4$. Then solve this equation for y, which is $y = -2$.

5. Sample: Substitute x and y into each equation. If both values satisfy both equations, then this verifies the answer.

6. $x = 3y - 2z + 11$

7. $y + 5z = 16$

8. $4y - 8z = -20$

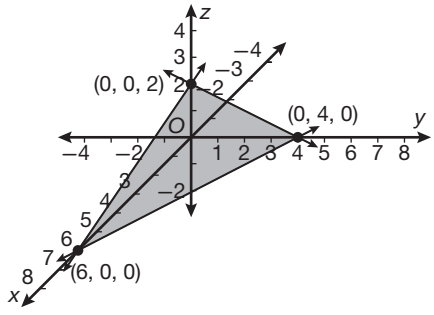
9. two; y and z

10. $y = 1$; $z = 3$

11. $x = 8$; By substituting the other two variables into one of the original equations

12. Set $y = 0$, set $z = 0$, and then solve for x.

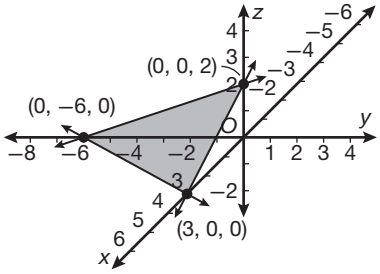
13. (6, 0, 0), (0, 4, 0), (0, 0, 2)

14. 

Investigation Practice 3

a. (2, −1, 1)

b. (0, 0, 2), (0, −6, 0), (3, 0, 0)



Lesson Activity 4

1. See student work.

2. See student work.

3. See student work.

4. 12 9 5 10 9 14
25 9 12 12 1 13 9

5. a. 8 16 1 7 10 1
14 17 5

b. 1 23 13 9 3 10 1

6. Check students' work for parts a–c.

d. QUIZ THIS FRIDAY

7. 14 4 6 12 9 5 10 9
14 25 9 12 12 1 13 9

8.
$$\begin{bmatrix} 14 & 12 & 10 \\ 4 & 9 & 9 \\ 6 & 5 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 12 & 9 \\ 9 & 1 & 0 \\ 12 & 13 & 0 \end{bmatrix}$$

9.
$$\begin{bmatrix} 34 & -44 & -12 \\ 22 & -31 & -19 \\ 34 & -48 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 43 & -52 & 8 \\ 9 & -9 & 13 \\ 12 & -12 & -41 \end{bmatrix}$$

10. 34 22 34 −44 −31
−48 −12 −19 15 43
9 12 −52 −9 −12 8
13 −41

11. The matrix product creates a message that cannot be hacked one letter at a time.

12. d. 14 15 17 12 25 9
12 12 1 13 9 17
12 12 9 5 16 10 9

e. THIS MESSAGE IS SECURE

Investigation Practice 4

a. THIS IS A CODED MESSAGE

b. THE PASSWORD IS MATRIX

Lesson Activity 5

1. Yes, because the four conditions are still met.

2. No, because condition 2 is not met (there are four possible outcomes).

3. No, because condition 2 is not met (there are six possible outcomes).

4. Yes, it is a binomial experiment.

Binomial Experiment	Does the Test Scenario Comply?
There are n trials in the experiment	Yes. There are 15 trials.
Only two possible outcomes per trial	Yes. The sum of the faces on the number cubes is greater than 3 or not.
Each trial is independent	Yes. The result of one trial does not influence the result of any other.
The probability of success is the same from trial to trial	Yes. $P(3 \text{ or less}) = \frac{2}{11}$ and $P(>3) = \frac{9}{11}$

There are 11 possible sums from 36 unique combinations.

5. a. Check students' work.

   b. Check students' work.

6. a. Check students' work.

   b. Check students' work.

   c. Answers will vary.

7. Check students' work.

8. Check students' work.

9.

Binomial Experiment	Does the Test Scenario Comply?
a. There are n trials in the experiment	Yes. There are 20 questions in the test.
b. Only two possible outcomes per trial	Yes. The guess is either correct or incorrect.
c. Each trial is independent	Yes. The result of one guess does not influence the result of any other guesses.
d. The probability of success is the same from trial to trial	Yes. Each guess has a 1 in 5 (or 20%) chance of success.

10. Answers will vary.

11. Check students' work.

e. Answers will vary.

f. Answers will vary.

Investigation Practice 5

a. binomial

b. Not binomial; conditions different for half the questions

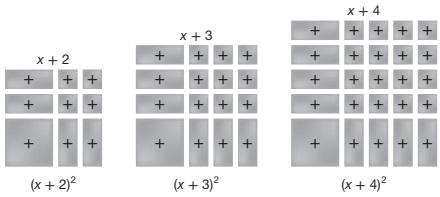
c. Check students' work.

Lesson Activity 6

1. Check students' work.

2. Check students' work.

3. a – c.



4.

Perfect Square	*b*	*c*
$(x+1)^2$	2	1
$(x+2)^2$	4	4
$(x+3)^2$	6	9
$(x+4)^2$	8	16

5. $\left(\frac{b}{2}\right)^2 = c$

6. a. For 6a, there should be *a* vertical line, one x^2 tile, 6 *x*-tiles, and one unit tile on the left and nothing on the right. The tiles should be arranged so they are three *x*-tiles on each of the two adjacent sides of the squared tile, so its obvious that eight unit tiles are missing.

   b. Eight 1-tiles.

   c. $x^2 + 6x + 9 = 8$; $(x+3)^2 = 8$

   d. $x = -3 \pm \sqrt{8}$

7. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

8. $x^2 + \frac{b}{a}x = -\frac{c}{a}$

9. $\left(\frac{b}{2a}\right)^2$

10. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

11. $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$

12. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

13. $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

14. $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{2a}}$

15. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

16. $x = \frac{-6 \pm \sqrt{6^2 - 4(2)(-3)}}{2(2)}$

$= \frac{-6 \pm \sqrt{36 + 24}}{4}$

$= \frac{-6 \pm \sqrt{60}}{4}$

$= \frac{-6 \pm 2\sqrt{15}}{4}$

$= \frac{-3 \pm \sqrt{15}}{2}$

Investigation Practice 6

a. perfect square

b. not a perfect square

c. perfect square

d. $x^2 + 4x = -1$

$x^2 + 4x + 4 = -1 + 4$

$(x + 2)^2 = 3$

$x = -2 \pm \sqrt{3}$

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16 - 4}}{2}$

$= \frac{-4 \pm \sqrt{12}}{2}$

$= \frac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$

e. $x^2 + 6x = -2$

$x^2 + 6x + 9 = -2 + 9$

$(x + 3)^2 = 7$

$x = -3 \pm \sqrt{7}$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)}$

$= \frac{-6 \pm \sqrt{36 - 8}}{2}$

$= \frac{-6 \pm \sqrt{28}}{2}$

$= \frac{-6 \pm 2\sqrt{7}}{2}$

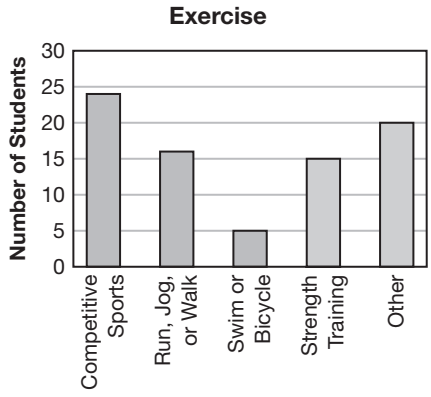
$= -3 \pm \sqrt{7}$

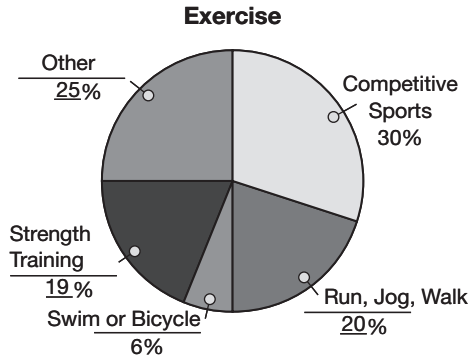
Lesson Activity 7

1. Answers will vary.

2. Answers will vary.

3. Yes

4. 

5. 

6. Biased; Possible explanation: The phrase "with most high school students" could influence a student to agree. Possible rewrite: Do you think parents give high school students too little freedom to make their own decisions?

7. Biased; Possible explanation: The phrase "unfair policy" could influence a student to not support the policy. Possible rewrite: Do you support the policy of requiring community service as a high school requirement?

8. Not biased

9. Answers will vary.

10. Systematic; Not biased

11. Self-selected, Biased; Possible explanation: The request for campaign contributions could result in a set of responses that is not a fair representation of the population.

12. Stratified; Not biased

13. Random; Not biased

14. Stratified, Probably biased; Possible explanation: It is not likely that there are approximately equal numbers of registered Democrats, Republicans, and Independents, so choosing 50 of each would probably be significantly out of proportion to their actual numbers.

15. Check students' work.

16. Population: all sophomores in the school; Sample: stratified, not biased

17. modes: 1.5, 2.0; median = 1.75

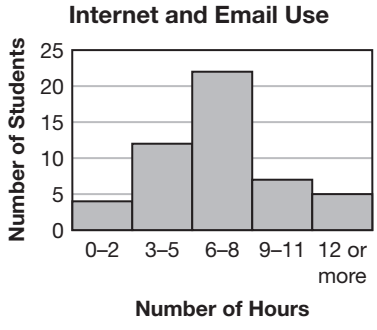
18. $5(2.0) + 3(2.5) + 1(3.0)$; 1.8

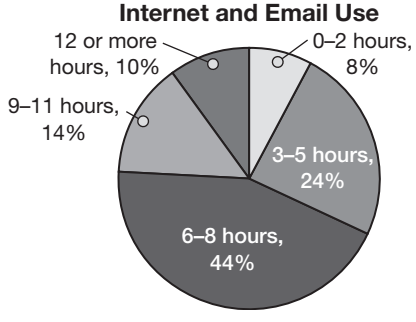
19. $5(2.0 - \overline{x})^2 + 3(2.5 - \overline{x})^2 + 1(3.0 - \overline{x})^2$; 0.535

20. 0.731

Investigation Practice 7

a.





b. Possible answer: There would be no way for a student to give a response such as 2.5 hours.

c. Possible answer: Students who emailed or used the Internet 3 hours per week could be represented in either the 0–3 bar or the 3–6 bar. A similar statement applies for 6, 9, and 12 hours.

d. Not biased

e. Biased; Possible explanation: The phrase "Knowing that teenagers tend to be irresponsible" could influence a student to answer no. Possible rewrite: Should people younger than 18 be allowed to drive without an adult in the vehicle?

f. Not biased

g. Biased; Possible explanation: The first sentence could influence a student to believe that recent climate changes are part of a natural long-term climate cycle rather than global warming caused by humans, and therefore believe that global warming is not a serious problem. Possible rewrite: Do you believe that global warming is a serious problem?

h. Self-selected; Not biased

i. Systematic, Biased; Possible explanation: Residents that live within a half mile of the proposed site could be more likely to not approve than residents who live farther away. The council members want to know if town residents approve, so the sample should be a fair representation of all residents in the town.

j. Stratified; Not biased

k. Random; Not biased

l. All students in the school; Stratified, not biased

m. mode $= 15$;
median $= 15$;
mean ≈ 15.46

n. $\sigma^2 \approx 1.582$

o. $\sigma^2 \approx 1.258$

p. Check students' answers.

Lesson Activity 8

1. 1 unit

2.

x	$f(x) = -2x^2 + 2x + 24$
0.5	24.5
1.5	22.5
2.5	16.5
3.5	6.5

3. 70 square units

4. 69.5 square units

5. The estimated answer is greater than the actual area.

6. Yes. Sample answer: If the widths of the rectangles are smaller, then the estimated area will be more accurate.

7. It represents the width of the rectangle.

8.

i	x_i	$f(x_i) = 0.5x^3 - x^2 + 2$
5	2.25	≈ 2.63
6	2.75	≈ 4.84
7	3.25	≈ 8.60
8	3.75	≈ 14.0

i	x_i	$f(x_i) = 0.5x^3 - x^2 + 2$
1	0.25	≈ 1.95
2	0.75	≈ 1.65
3	1.25	≈ 1.41
4	1.75	≈ 1.62

9. 18.5 square units

Investigation Practice 8

a. 56.25 square units

b. Total Area $= \left(\sum_{i=1}^{10} f(x)\right)$

c. 1530 square units

Lesson Activity 9

1. a. $1.00

   b. $1.00

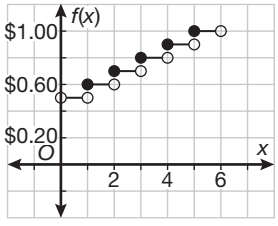
   c. $0.50

2. Yes; although 30 seconds is rounded down to 0 minutes, there is still a connection fee.

3. No; $5\frac{1}{4}$ rounded down to the nearest whole number is 5 minutes.

4.

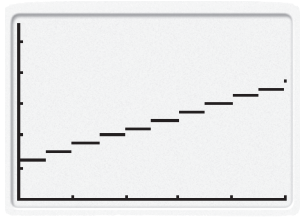
x	$f(x)$
$0 < x < 1$	$0.50
$1 \leq x < 2$	$0.60
$2 \leq x < 3$	$0.70
$3 \leq x < 4$	$0.80
$4 \leq x < 5$	$0.90
$5 \leq x < 6$	$1.00

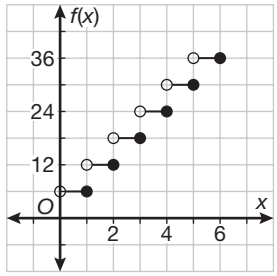


5. a. $f(x) = 0.10x + 0.50$

   b. Replace the variable, x, with $[x]$.

   c. $f(x) = 0.10[x] + 0.50$

   d. 

6. a. 

   b. $12, $12, $6

   c. $6

x	$f(x)$
$0 < x \leq 1$	$6
$1 < x \leq 2$	$12
$2 < x \leq 3$	$18
$3 < x \leq 4$	$24
$4 < x \leq 5$	$30
$5 < x \leq 6$	$36

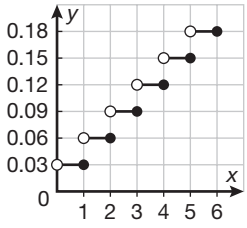
Investigation Practice 9

a. $f(x) = 0.03\lceil x \rceil$

b. domain: $x > 0$

c. least integer function; the number of ounces is rounded up to the nearest ounce.

d.



x	$f(x)$
$0 < x \leq 1$	0.03
$1 < x \leq 2$	0.06
$2 < x \leq 3$	0.09
$3 < x \leq 4$	0.12
$4 < x \leq 5$	0.15
$5 < x \leq 6$	0.18

e. $f(x) = 2\lceil x \rceil$

f. least integer function

x	$f(x)$
$-2 < x \leq -1$	-2
$-1 < x \leq 0$	0
$0 < x \leq 1$	2

Lesson Activity 10

4. The value of r increases as θ increases.

5. The curvature of the spiral increased.

6. The curvature of the spiral decreased.

7. The coefficient a determines the curvature of the spiral and b is the translation of the graph along the horizontal axis.

8. The number of petals is twice the value of b.

9. The number of petals equals the value of b.

10. $b = 10$

11. 19 petals

12. The length of the petals increases.

13. The length of the petals decreases.

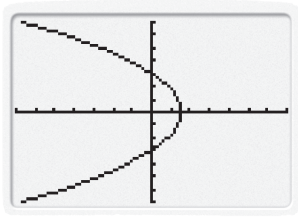
14. a circle

15. an ellipse

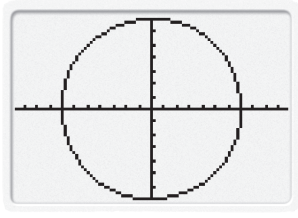
16. a parabola

17. the radius of the circle

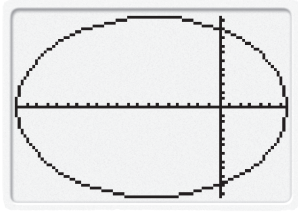
18. Sample:
$r = \pm \frac{3}{1 + \cos(\theta)}$



19. Sample: $r = \pm 7$ or $r = 14\sin(\theta)$



20. Sampler:
$r = \frac{10}{1 + 5\cos(\theta)}$



Investigation Practice 10

a. Sample: $r = 3 \sin \theta$

b. The curvature of the spiral decreased.

c. $y = \sin(7\theta)$

d. $y = \sin(9\theta)$

e. Sample: $y = 4 \sin(3\theta)$

f. ellipse

g. parabola

Lesson Activity 11

1. See students' graphs.

2. **a.** $\theta = 45°$

b. $\theta \approx 115.57°$

c. $\theta \approx 288.43°$

d. $\theta \approx 104.03°$

e. $\theta \approx 291.80°$

f. $\theta \approx 165.96°$

3.

a	b	$r = \sqrt{a^2 + b^2}$
4	4	5.6569
−7	14	15.6525
3	−9	9.4868
−2	8	8.2462
2	−5	5.39
−4	1	4.12

4. **a.** $z = 4.47 \cos(333.43)° + i4.47 \sin(333.43)°$

b. $z = \sqrt{2} \cos(225°) + i\sqrt{2} \sin(225°)$

c. $z = 5 \cos(53.13°) + i5 \sin(53.13°)$

5. **a.** $\frac{7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}i$

b. $10i$

c. $-2.5 + \frac{5\sqrt{3}}{2}i$

d. $-15.97 - 5.81i$

6.

r	θ	a	b
7	45°	4.95	4.95
10	90°	0	10
5	120°	−2.5	4.33
17	200°	−15.97	−5.81

7.

$a + bi$	r	θ	Polar Form
$4 + 6i$	7.2	56.31°	$7.2 \cos(56.31°) + i7.2 \sin(56.31°)$
$7 + 7i$	9.90	45°	$9.90 \cos(45°) + i9.90 \sin(45°)$
$-3 + 4i$	5	126.87°	$5 \cos(126.87°) + i5 \sin(126.87°)$
15	15	0°	$15 \cos(0°) + i15 \sin(0°)$
$28i$	28	90°	$28 \cos(90°) + i28 \sin(90°)$

8. a. $171.5 + 297.05i$

   b. $9i$

   c. $-8 + 13.86i$

   d. $-i$

   e. 16

   f. -64

9. a. $125\cos(159.39°) + i125\sin(159.39°)$

   b. $40{,}000\cos(180°) + i40{,}000\sin(180°)$

   c. $13^6\cos(44.28°) + i13^6\sin(44.28°)$

   d. $4^8\cos(0°) + i4^8\sin(0°)$

   e. $7^{12}\cos(0°) + i7^{12}\sin(0°)$

Investigation Practice 11

a.

$a + bi$	r	θ	Polar Form
$5 + (-5i)$	7.07	315°	$7.07\cos(315°) + i7.07\sin(315°)$
$-7 + 10i$	12.21	124.99°	$12.21\cos(124.99°) + i12.21\sin(124.99°)$
$0 + 18i$	18.00	90°	$18\cos(90°) + i18\sin(90°)$
$25 + 0i$	25.00	0°	$25\cos(0°) + i25\sin(0°)$
$-9 + (-3i)$	9.49	198.43°	$9.49\cos(198.43°) + i9.49\sin(198.43°)$

b. $2809\cos(296.22°) + i2809\sin(296.22°)$

c. $64{,}057{,}682.83\cos(120.79°) + i64{,}057{,}682.83\sin(120.79°)$

d. $10^8\cos(65.04°) + i10^8\cos(65.04°)$

e. $9^{12}\cos(0°) + i9^{12}\sin(0°)$

f. $19^{15}\cos(270°) + i19^{15}\sin(270°)$

Lesson Activity 12

1. Check students' work.

2. As n gets larger in value, the number of calculations is still the same.

3. Check students' work.

4. $\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$

5. $a_{m+1} = m + 1$

$$\left(\sum_{k=1}^{m} k\right) + a_{m+1}$$

$$= \frac{m(m+1)}{2} + (m+1)$$

$$= \frac{m^2 + m}{2} + \frac{2m+2}{2}$$

$$= \frac{m^2 + 3m + 2}{2}$$

$$= \frac{(m+1)(m+2)}{2}$$

6. $\sum_{k=1}^{m+1} k = \frac{(m+1)((m+1)+1)}{2}$

$$= \frac{(m+1)(m+2)}{2}$$

$$\sum_{k=1}^{m+1} k = \left(\sum_{k=1}^{m} k\right) + a_{m+1}$$

7. Yes

8. $a_k = 2k$

9. $\sum_{k=1}^{n} 2k$

10. $n(n+1)$

11. Check students' work.

12. a. Check students' work.

    b. $m(m+1)$

    c. $m(m+1) + 2(m+1)$
    $= m^2 + m + 2m + 2$
    $= m^2 + 3m + 2$
    $= (m+1)(m+2)$
    $= (m+1)$
    $((m+1)+1)$,
    which maintains the form of the original expression.

Investigation Practice 12

a. Check students' work.

b. $a_k = 2k - 1$

c. $\sum_{k=1}^{n}(2k-1)$.

d. n^2

e. Check students' work.

f. m^2

g. $(m+1)^2 + 2(m+1) + 1$
$= m^2 + 2m + 1 + 2m + 3$
$= m^2 + 4m + 4$
$= (m+2)^2$
$= ((m+1)+1)^2$, which maintains the form of the original expression.

h. The formula is true.

i. Incorrect formula,; The correct formula is $\frac{5n(n+1)}{2}$.

j. $\sum_{k=1}^{n} 15k = \frac{15n(n+1)}{2}$;
Sample: The LCM of 3 and 5 is 15. To find the formula for the sum of consecutive numbers divisible by 3 and 5, find the formula for the sum of consecutive numbers divisible by 15:
$\sum_{k=1}^{n} 15k = \frac{15n(n+1)}{2}$.